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VOLUME I

ULTRASOUND PHYSICS & INSTRUMENTATION

4TH EDITION

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Dedication

This book is dedicated to my wife Carol, my mother Graziella, my three-year-old daughter Gina Luisa, my twenty-month-old daughter Cristiana Graziella, and in loving memory of my grandmother Gina Luisa: four generations of women who have shaped my life.

Of course I cannot forget my twenty-month-old son Franco Raffaello whose hugs redefine affection, or my eldest son Paul (forever Paulie to me), who loves to sit in on lectures and "help out." Finally, I must acknowledge my father Frank, whose tireless work and selflessness afforded me educational opportunities for which I feel blessed.

A special thanks to my friends and colleagues who took the time and made the effort as section reviewers and who provided quality images. Your time, suggestions, and expertise are much appreciated.

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and special thanks to Carol, Debbie, Eric, Halan, and Monica whose tireless efforts helped bring this book to fruition.

Preface

The greatest challenge with writing a textbook is that every reader brings a different level of knowledge, experience, and goals with him or her. For some people, this book represents their first exposure to ultrasound, and the book is to serve as a foundation on which to build. Others bring twenty or more years of experience and use this book as a way reinforcing the principles on which they perform their daily scans, or as a means to better understand new ultrasound techniques. Some are using this book in an ultrasound educational program, and others are using this book to prepare for a credentialing exam. Add to these differences in experience and goals the fact that some people experience dramatic feelings of fear and loathing just from hearing the word "physics" and you have a real conundrum when determining how to structure a textbook.

It should now be clear that the myriad goals and levels of experience make this a very challenging book to write. No one book can be everything to everyone. So instead, this book is really three books in one. Topics are divided into levels so that different level students can progress at a pace appropriate for their background, experience, and goals. Beginning students can follow Level 1 throughout the book, leaving Level 2 and Level 3 for when they have more ultrasound experience. More advanced students can choose to skip Level 1 and go right to Level 2, or use Level 1 as a refresher and use Level 2 as a means of preparing for the credentialing exams and advancing their knowledge. Level 3 is intended for those readers who really want to be challenged, or for content that is perhaps outside the areas generally tested on the credentialing exams. Extensive "Keypoints" sections are included at the conclusion of each chapter to both integrate concepts and serve as a study guide. The following is a description of each level:

Level 1: **Ultrasound Physics Basics**
Level 1 material focuses on the underlying physics and basic concepts critical for developing skill in the use of diagnostic ultrasound. Level 1 presumes no knowledge other than the basic abilities that come from general schooling. This level also serves as a good refresher for people who have good ultrasound experience but weaker backgrounds in physics and basic mathematics.

Level 2: **Exam Level Ultrasound Physics**
Level 2 material covers basic topics often outlined on the credentialing exams. Furthermore, Level 2 material is intended to generate a more profound understanding of the concepts so that the relationship of the physics fundamentals to the quality of the diagnostic ultrasound is understood. In other words, understanding Level 2 should not only prepare you for your board exams, but also result in better patient care.

Level 3: **Advanced Ultrasound Physics Concepts and Applications**
Level 3 material contains advanced topics, newer ultrasound techniques, or even just higher level material for those people who want to be challenged. At times, Level 3 will also contain specific applications of the physics to a specialty area such as cardiac, vascular, or general ultrasound.

Keypoints:

The keypoints serve as a chapter-by-chapter review of the fundamental principles. These sections serve both as a means of highlighting the main points as well as an exam review. Many of the keypoints included serve as the basis for exam questions, and as such should be reviewed by all exam candidates.

The Importance of Understanding the Structure of this Book

There are three reasons why understanding the structure of this book is so important:

1. So that you can customize how you use the book to your experience level and goals.
2. So that you have a clear indication of when you are knowledgeable to take a credentialing exam.
3. So that you have a systematic approach to increase your knowledge and clinical abilities from your current state.

In addition to serving as a core for an ultrasound physics program and a reference for your laboratory, this textbook has also been designed as an independent learning program to assist candidates preparing for their credentialing and board exams. Volume II, Appendix C contains a comprehensive Test Taking Strategies section to help improve your test taking skills including specific approaches detailing the incorporation of logic and reasoning skills for multiple choice exams. This section should be reviewed before and after reading the text. Volume II, Appendix M contains study suggestions, a study guide, and instructions for obtaining continuing medical education credits.

A Final Word about this Book's Structure:

There are many ultrasound physics books that either overshoot or undershoot the intended goals of the reader. In addition, there are books that are very easy to read, but stop short of building the knowledge necessary to demonstrate competence on a credentialing exam. More importantly, these books fall short of imbuing the reader with the knowledge necessary to improve patient care. There are other books that, although technically excellent, presume too much knowledge for most people such that the reader feels as if they are drowning from the very first page. By writing this book in three different levels, I am hoping to reach out to a wider audience, increasing knowledge for both the experienced and the neophyte in ultrasound. I have chosen to create a book that takes students through the first level and beyond with a clear path to the knowledge necessary to demonstrate competency at the credentialing exam level. I hope I have written a book that does not presume so much knowledge that students become overwhelmed and are afraid to utilize the text, but on the other hand pushes and challenges the student to continue learning. In essence, I have tried to write this book so that each level becomes appropriate as the reader's knowledge grows.

I believe the first step to knowledge is a true assessment of where you are, where you want to be, and what path you are willing to take to get there.

And so starts the journey …

Pegasus Lectures Physics and Instrumentation
Independent Learning Program

Jointly Sponsored by A. Webb Roberts Center for Continuing Medical Education of Baylor Health Care System, Dallas and Pegasus Lectures, Inc.

CME ENDURING MATERIAL INFORMATION

Faculty

Frank Miele, MSEE
President, Pegasus Lectures, Inc.

Frank graduated cum laude from Dartmouth College with a triple major in physics, mathematics, and engineering. While at Dartmouth, he was a Proctor Scholar and received citations for academic excellence in comparative literature, atomic physics and quantum mechanics, and real analysis. After completing his graduate work, Frank was awarded the Ruth Goodrich Prize for Academic Excellence. After co-teaching a course in digital electronics at Dartmouth, Frank was a research and design engineer and project leader, designing ultrasound equipment and electronics for more than ten years. In that role, Frank designed the hardware for the first parallel processing color Doppler system, created a Doppler system platform, designed HPRF Doppler, created the first released adaptive ultrasound processing technique, designed transcranial Doppler and transcranial imaging, worked on multiple transducer project teams, and performed extensive clinical trial testing and research.

Frank has been the vice president of Research and Development and chief scientist for a medical device company investigating ultrasound related hemodynamic based measurements. As a researcher and designer of ultrasound, he has lectured across the country to sonographers, physicians, engineers, and students on myriad topics. Frank has authored the Ultrasound & Physics Instrumentation Independent Learning Program, produced multiple educational videos, created exam simulation programs, as well as created the analysis algorithm method and apparatus for evaluating educational performance (patent pending). Frank has served as an author and Co-Chief editor for the ASCeXAM Simulation Review CD in conjunction with the American Society of Echocardiography. He has also served on the faculty for the Society of Vascular Ultrasound and Society of Vascular Surgery and is credited with several ultrasound and medical device patents, trade secrets, and publications.

Purpose and Target Audience

This activity is designed to familiarize physicians and sonographers with the physics and instrumentation concepts employed in diagnostic ultrasound and provide a method to prepare for ultrasound physics credentialing exams and/or accreditation. It will be of interest to, but not limited to, radiologists, cardiologists, neurologists, vascular surgeons, cardiovascular surgeons, anesthesiologists, and/or physicians providing interpretation of diagnostic ultrasound, preparing for accreditation, and/or desiring to improve their understanding of ultrasound physics.

Medium

Printed text
CD

Objectives

Upon completion of the activity, the participant should be able to:

- Define areas of strengths and weaknesses in their understanding of ultrasound physics.
- Comprehend the effect of system controls and transducer parameters on the diagnostic quality of an ultrasound image.
- Demonstrate improved understanding of ultrasound physics and how physics can affect the integrity of a diagnostic image.
- Demonstrate improved interpretative skills for diagnostic ultrasound, Doppler and hemodynamic variables.
- Demonstrate improved preparation for the ultrasound physics credentialing exam.

CME Credit

This activity has been planned and implemented in accordance with the Essential Areas and Policies of the Accreditation Council for Continuing Medical Education (ACCME) through the joint sponsorship of the A. Webb Roberts Center for Continuing Medical Education of the Baylor Health Care System, Dallas and Pegasus Lectures, Inc.

The A. Webb Roberts Center for Continuing Medical Education of Baylor Health Care System, Dallas designates this educational activity for a maximum of 35 Category 1 credits toward the AMA Physician's Recognition Award. Each physician should claim only those credits that he/she actually spent in the activity.

The A. Webb Roberts Center for Continuing Medical Education of Baylor Health Care System, Dallas is accredited by the ACCME to provide continuing medical education for physicians.

Faculty Disclosure

Frank Miele is the president of Pegasus Lectures, Inc. and owner of Miele Enterprises, LLC.

No unlabeled or investigational uses of a product or medical device are addressed in this CME activity.

Instructions

Participants must:
1. Read Volumes I & II of Ultrasound Physics and Instrumentation
2. Complete conceptual questions and exercises within Volumes I & II
   (Refer to study suggestions in Vol. II, Appendix M)
3. Complete the final exam and evaluation

Final Exam, Evaluation and Processing Fee

At the conclusion of this activity, participants must complete the final exam and the evaluation. A completed evaluation form must accompany the final exam. Please indicate on the evaluation form if you are applying for AMA/PRA Category 1 credit or SDMS CME credit and if you are a U.S. licensed physician or not. There is a $30 processing fee for AMA/PRA Category 1 credit. **The check for AMA/PRA credit should be made payable to: A. Webb Roberts Center.** Participants should send the original final exam for scoring (no copies accepted), the processing fee, the evaluation and any correspondence to:

Pegasus Lectures, Inc.
PO Box 157
Forney, TX 75126

Tel: 972-564-3056
Fax: 972-552-9186

NOTE: Participants must achieve a **75% pass rate** on the final exam to be awarded AMA/PRA CME credit. Feedback on exam scores will be provided. A CME certificate will be mailed directly to the participant from the A. Webb Roberts Center for Continuing Medical Education of Baylor Health Care System, Dallas.

Date of Original Release: May 2001
Date of Most Recent Update: October 2005

Estimated time to complete the educational activity: 35 hours

This independent learning educational activity has been approved for 35.0 hours of SDMS CME credit. Each participant should claim only those hours of credit that he/she actually spent in the educational activity.

At the conclusion of this activity participants should complete the final exam and evaluation. A completed evaluation form must accompany the final exam. *Please indicate on the evaluation form if you are applying for AMA/PRA Category 1 credit or SDMS CME credit*. Participants should send the final exam, evaluation and any correspondence to:

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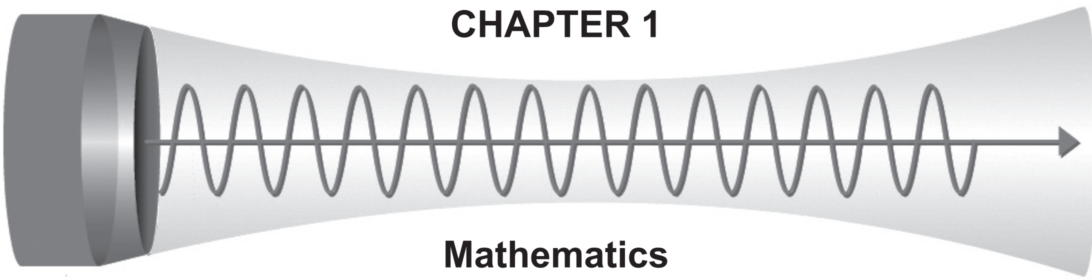

CHAPTER 1

Mathematics

Introduction

Without math, it is not possible to learn physics. Math is to physics what a paintbrush is to a painter or physical conditioning is to an athlete. The foundation of physics is mathematics. Therefore, to understand physics, and not just memorize facts, you must have at least a rudimentary understanding of mathematics.

Types of Math

Most people think of mathematics in terms of basic arithmetic and numerical manipulation. Whereas, this is certainly true, it is a very narrow view of mathematics. Mathematics is really an enormous field that includes many different topics. The field of mathematics includes many disciplines such as algebra, number theory, geometry, calculus, trigonometry, topology, and even logic and reasoning. Although you will not need to become an expert in any of these disciplines, the more you learn in mathematics, the easier it will be to understand physics.

How Much Math Will You Need?

Many people in the field of ultrasound will state that there is very little mathematics on the board exams. I do not agree. I think this disparity in opinions stems from the very narrow definition of mathematics as numerical calculations. It is true that there are very few numerical calculations on most of the credentialing exams, and certainly never such intensive calculations such that one would require a calculator. However, depending on the specific credentialing test and the test version, there are approximately twenty-five to forty percent of the questions that involve some form of mathematics.

As you will discover throughout this book, the mathematics included on the credentialing exams generally does not include performing many calculations, but rather asks relative relationships and logical conclusions from the mathematical relationships between variables. In other words, instead of asking you to calculate the resistance to flow for a fluid flowing through a vessel, you may be asked how the resistance to flow will change with changes in parameters that define the vessel. Instead of asking you to calculate a Doppler shift for a given transducer frequency, given a specific angle to flow and a blood flow velocity, you may be asked how the Doppler shift would change if a different transducer operating frequency were used given the same angle to flow and blood flow velocity. Answering this type of "relative" question involves math skills, which many students have not used for a long period of time, or worse, never developed. This last point is precisely why it is so critical for you to learn the basic mathematics, as outlined below.

To learn the basics of ultrasound (Level 1) you will need a proficiency in the basic mathematical functions. Specifically, you will need to:

- Be comfortable with the language of mathematics and translating English into mathematical functions.
- Add, subtract, multiply, and divide.
- Deal with fractions, percentages, and decimal notation.
- Understand exponential form and become fluent with the metric system.
- Understand the concept of reciprocals.
- Understand basic relationships of variables within an equation (proportionality and inverse).
- Perform algebraic manipulation of equations.

To master Level 2, you will need some higher-level math skills such as:

- Understand the difference between absolute and relative information.
- Understand the difference between linear and non-linear relationships.
- Recall or determine the equations commonly used in ultrasound physics and in hemodynamics.
- Understand the basic trigonometric functions of the sine and the cosine.
- Understand and apply the concepts of logarithms and decibels.
- Understand the basics of the binary system (relative to base 10).

To master Level 3, you will not necessarily need to develop many more math skills than suggested for Level 2. However, Level 3 presumes a more fluid working knowledge of the mathematics needed to master Level 2. The few additional mathematical topics that might help with Level 3 but are not necessarily required are:

- Some basic calculus.
- Understand rates of change (derivatives).
- Understanding integration.

How to Learn Mathematics

The good news is I have never encountered a student unable to master the mathematics necessary to pass the examination. The bad news is that learning this mathematics requires a structured approach, time, work, and patience.

A Structured Approach, Time, Work and Patience

Adults don't learn the same way children learn. Children tend not to be afraid of making a mistake in front of their peers. Adults, in comparison, live in fear and dread that someone will recognize that they are ignorant of even the slightest detail. I believe this approach of learning in fear puts an extraordinarily heavy and unfair burden upon adults in the position of student.

There is no way that anyone will understand everything just by reading the material once. Expecting to understand immediately is not only unreasonable, but puts a tremendous stress on the student. If you realize that learning is a process that only comes slowly over time through work and patience, you will not panic when something isn't clear the first time. It is not fair to yourself to lose self-esteem because you don't fully understand a concept the first time through the material. I often use the analogy that learning is like building a house. Before you can get to the fun part of decorating the interior with intricate art and furniture, you have to go through the backbreaking work of digging a hole in the ground, setting up forms, and building a foundation. Without a solid foundation, the house will never stand.

It is time to dig the hole and do the work necessary so that you can build the foundation.

1. Mathematic Basics

1.1 Numbers

In mathematics, there are many categorizations which group numbers together based on their similar properties. For example, there are counting numbers (the natural numbers), negative counting numbers (the negative natural numbers), the set of all the natural numbers, negative numbers and 0 (called the integers), numbers which can be expressed as the ratio of two integers (rational numbers), and numbers which cannot be expressed as the ratio of two integers (irrational numbers), etc.

Natural Numbers: 1, 2, 3, 4, …
Negative Natural Numbers: -1, -2, -3, -4, …
Integers: -5, -4, -3, -1, 0, 1, 2, 3, 4, 5, …
Rational Numbers: (all numbers which can be expressed as p/q where p and q are integers)
Irrational Numbers: (all numbers which cannot be expressed as p/q where p and q are integers)

For ultrasound physics, you will not need to know precise definitions of all of the various classifications of numbers. What you will need is a general ability to work with numbers including the basic mathematical operations of addition, subtraction, division, and multiplication.

1.2 Basic Mathematical Notation (symbols used in basic mathematics)

Addition	+	
Subtraction	-	
Multiplication	x	Example: m x f implies the variable m multiplied by the variable f.
	*	Example: t * v implies the variable t multiplied by the variable v.
	•	Example: c • v implies the variable c multiplied by the variable v.
	( )	Example: 3(7) implies the number 3 times the number 7.
No symbol		Example: 3z implies the number 3 multiplied by the variable z.
Division	/	Example: m / f implies the variable m divided by the variable f.
	÷	Example: j ÷ k implies the variable j divided by the variable k.
Equality	=	

Inequalities:

Greater than	>	Example: g > 3 is read as g is greater than the number 3.
Less than	<	Example: h < 6 is read as h is less than the number 6.
Greater than or equal to	≥	Example: k ≥ r is read as k is greater than or equal to r.
Less than or equal to	≤	Example: h ≤ 6 is read as h is less than or equal to 6.

*Note: Children generally learn to use the letter "x" to stand for multiplication. Once students reach algebra, there is generally a shift that occurs such that symbols other than "x" are often used to denote multiplication. This shift occurs since the letter x is generally used to stand for the unknown quantity in an algebraic expression. Since it is easy to confuse the "x" that stands for multiplication with the "x" that stands for a variable, other symbols become more commonly used. Therefore, there are many symbols used to indicate multiplication, such as: x, •, *, and sometimes ( ). All of these symbols will be used throughout this text and interchanged freely so as to accustom you to each of these notations.*

1.3 Basic Mathematical Definitions

Constant:	A number which cannot change (Example: 3, 7, -14, 6 are all constants).
Natural constant:	A number which reoccurs naturally in the universe in relation to a specific parameter (Example: pi (π) for circles).
Coefficient:	A constant term used as a multiplier of a variable (Example: in the expression $7z^2$, the number 7 is the coefficient for the variable term z^2).
Variables:	A physical quantity which can vary or change (Example: in the expression $3x^2$, the variable is represented by the letter x).

1.4 The Value of Estimating

The ability to estimate quickly is very handy in every day life. Learning to make good estimates comes from practice and a little bit of thinking. For example, if you were asked to solve the problem what is 19 times 20, and you were not allowed to use a calculator or paper and pencil, what would you do.

Approach 1: (rounding off: estimation)

Find a way of rounding off the numbers into two numbers you can easily multiply in your head. For example 19 x 20 is a little less than 20 x 20. Since 20 x 20 is 400, your first answer would be just a little less than 400.

Approach 2: (actual answer using estimation to simplify the math)

Start with Approach 1 and add one more step. Since 19 x 20 can be written as (20-1) x 20 which is the same as (20 x 20) – (1 x 20), you can actually solve this problem exactly in your head. As you solved in Approach 1, 20 x 20 is 400. Since the correct answer is actually 20 less than 400, the answer is 380.

In terms of ultrasound physics and hemodynamics, there are times when you should estimate the answer to a problem to make certain that you have not made a simple math calculation error. The best way to develop this ability is to put away the calculator and start practicing calculating and estimating in your head.

1.5 Exercises: Estimating

1. What is 24 x 6? (calculate both a rounded off answer and the actual answer)

2. What is 249 x 3? (calculate both a rounded off answer and the actual answer)

3. What is 12 ÷ 3.1? (calculate a rounded off answer only)

4. What is 199 ÷ 5? (calculate a rounded off answer only)

5. What is 37 x 11? (calculate both a rounded off answer and the actual answer)

2. Fractions, Decimal Form, and Percentages

One of the skills in mathematics that we learn earliest is how to deal with fractions and percentages. Unfortunately, the use of calculators has, for most people, caused this skill to deteriorate. Being able to deal with fractions and percentages is critical in physics and medicine.

A fraction consists of two parts: a number on top called the numerator, and a number on the bottom called the denominator. An increase in the numerator with no change to the denominator results in an increase in the fraction (see proportionality in Section 10.1).

For the fraction defined as: $Fraction = \frac{p}{q}$.

*An increase in p implies an increase in $\frac{p}{q}: \left(\text{if } p \uparrow \Rightarrow \frac{p}{q} \uparrow \right)$.

Conversely, an increase in the denominator with no change to the numerator results in a decrease in the fraction (see inverse proportionality in Section 10.3).

For the same fraction defined as: $Fraction = \frac{p}{q}$.

An increase in q implies a decrease in $\frac{p}{q}: \left(\text{if } q \uparrow \Rightarrow \frac{p}{q} \downarrow \right)$.

Often, a fraction is not written in its simplest form, implying that there is a multiplying factor which is common between the numerator and the denominator. In these cases, the fraction can be "simplified", or reduced to "simplest form" by dividing both the numerator and denominator by the common multiple.

◊ ***Examples:***

$$\frac{4}{8} = \frac{4*1}{4*2} = \frac{4}{4}*\frac{1}{2} = 1*\frac{1}{2} = \frac{1}{2}$$

$$\frac{14}{200} = \frac{2*7}{2*100} = \frac{2}{2}*\frac{7}{100} = 1*\frac{7}{100} = \frac{7}{100}$$

$$\frac{120}{1200} = \frac{120*1}{120*10} = \frac{120}{120}*\frac{1}{10} = 1*\frac{1}{10} = \frac{1}{10}$$

**Note: the symbol (⟹) stands for the word "implies."*

Additionally, all fractions can be written in decimal form and as percentages. Converting from fractions to decimal form is simply the process of division. Converting from decimal form to percentages is just multiplication by 100%.

◊ ***Examples:***

$$\frac{1}{1} = 1 = 100\% \qquad \frac{1}{2} = 0.5 = 50\%$$

$$\frac{1}{3} = 0.333 = 33.3\% \qquad \frac{1}{4} = 0.25 = 25\%$$

$$\frac{2}{1} = 2 = 200\% \qquad \frac{5}{2} = 2.5 = 250\%$$

Another way of thinking of fractions is how many times something occurs per hundred events. As such, it is easy to convert fractions to percentages when the denominator is a factor of 10. (You should notice that this process is equivalent to counting the number of decimal point shifts.)

◊ ***Examples:***

$$\frac{7}{100} = 0.07 = 7\% \qquad \frac{2}{10} = \frac{20}{100} = 0.2 = 20\%$$

$$\frac{43}{1000} = \frac{4.3}{100} = 0.043 = 4.3\% \qquad \frac{16}{10} = \frac{160}{100} = 1.6 = 160\%$$